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Relativistic description of proton–proton bremsstrahlung

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Abstract

We investigate the influence of negative-energy states in proton–proton bremsstrahlung in a fully relativistic framework using the t-matrix of Fleischer and Tjon. The contribution from negative-energy states in the single-scattering diagrams are found to be large, indicating that relativistic effects are sizable. The rescattering contribution compensates some of the effect, but at higher photon energies we find that the relativistic contributions become increasingly more important. The cancellation found at lower energies is shown to be due to a low-energy theorem. © 1997 Elsevier Science B.V.

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Proton–proton bremsstrahlung [1] is one of the most simple processes involving the half off-shell NN interaction. Since protons are equally charged particles, electric dipole radiation is suppressed and higher-order effects play an important role. Thus we can get information on the NN force not easily obtained from other processes. At lower photon energies we should in general expect that negative-energy-state contributions to the bremsstrahlung process should be suppressed due to the low-energy theorem [2]. At higher energies however negative-energy states can contribute in a significant way. In this Letter we investigate the role of negative-energy states using a relativistic model that includes these states in a dynamical way. We find that including these states does indeed give substantial effects in both the cross section and the analyzing power. As compared to recent work by Eden and Gari [3] who used a Hamiltonian formalism, and of de Jong and Nakayama [4], who used the NN t-matrix of a relativistic spectator model [5], the relativistic contributions are in general found to be more enhanced, especially in the cross-section predictions. The effects are of the order of 20% in the cross section for forward and backward photon angles and small proton angles (i.e. large photon momenta).

Before describing our full analysis, we first present a simple order-of-magnitude argument to show that negative-energy states may be important in bremsstrahlung. The standard argument that the contributions from the negative-energy states are suppressed is that the propagator S_ρ (ρ being an index labeling the energy state) is smaller for negative-energy states due to the energy denominator. At sufficiently high photon energy the single-scattering emission diagrams are expected to dominate. Consider for example the case where a photon with momentum q is emitted prior to the NN interaction, Fig. 1a. The positive- and negative-energy parts $S_\pm$ of the propagator of the intermediate nucleon with momentum $p-q$ behave like

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$S_+^{-1} = E_p - E_{p-q} \approx pq/E_p$ and $S_-^{-1} = E_p + E_{p-q} \approx 2E_p$. Hence the negative-energy states are expected to be suppressed by a factor of pq/E_p^2. However, we have also to account for the current matrix elements in the diagram. Let us for simplicity assume that the NNγ vertex is given by $e\gamma^\mu$. The parts of the photon coupling probed by bremsstrahlung are the transversal components, which clearly have large off-diagonal ρ-spin matrix elements. For the case of intermediate positive-energy states the dominant contributions are coming from the large and small components of the initial and intermediate nucleon spinors

$$\bar{u}_+(p')\gamma^\mu u_+(p) \propto \frac{p'}{E'+m} \pm \frac{p}{E+m} \approx \frac{p}{m}, \qquad (1)$$

where we have effectively taken the static limit, neglecting terms of order q/m to the e.m. vertex operator. Assuming that the photon momentum is small compared to the energy of the proton, we have $E' \approx E \approx m$. For intermediate negative-energy states we get

$$\bar{u}_-(p')\gamma^\mu u_+(p) \propto 1 \pm \frac{pp'}{(E'+m)(E+m)} \approx 1. \qquad (2)$$

From this we see that the overall suppression factor becomes only of the order of $(pq/m^2) \times (p/m)^{-1} \approx q/m$. Since the matrix elements between positive- and negative-energy states in the t-matrix are found empirically in relativistic models to be of the same order in magnitude as that between positive-energy states, we see that the total suppression of negative-energy contributions is approximately of the same size as the q-dependent corrections to the static limit of the NNγ-vertex in Eq. (1). These q-dependent terms are in general not negligible. Thus we expect negative-energy states to give a significant contribution, a situation comparable to Compton scattering, where for low photon energies one finds the Thomson limit being essentially given by the z-graph contribution [6].

For a complete calculation of the various bremsstrahlung contributions shown in Fig. 1 a relativistic NN interaction is needed. To generate this we take the Bethe–Salpeter equation as a starting point,

$$T(p',p;P) = V(p',p;P) - \frac{i}{(2\pi)^4}\int d^4k\, V(p',k;P)S_2(k,P)T(k,p;P), \qquad (3)$$

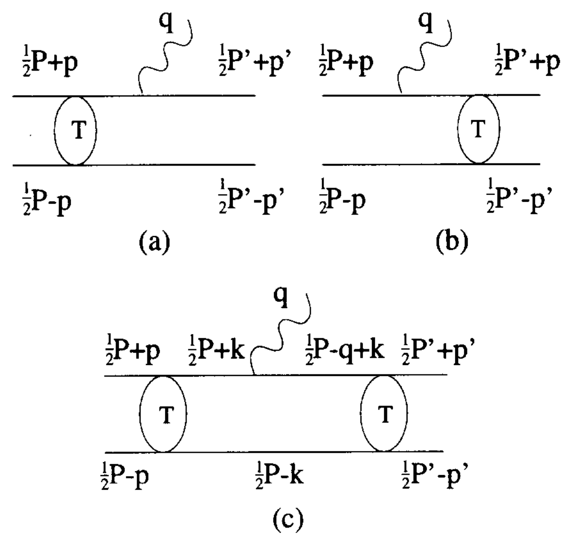

Fig. 1. The contributing diagrams occurring in the present calculation with definitions of the momenta. Diagrams (a) and (b) are the single-scattering diagrams, diagram (c) is the rescattering diagram.

where S_2 is the free two-nucleon propagator and V is the interaction kernel. The momenta p and P are the relative and total momentum of the two nucleons. We use the relativistic one-boson-exchange (OBE) model of Refs. [7,8] where the interaction is described by the exchange of π (with a pure pseudovector coupling), ρ, ω, η, ϵ (or σ) and δ mesons. A phenomenological cutoff by means of a monopole form factor depending only on the meson 4-momentum is introduced at each meson–nucleon vertex to ensure convergence. Although the equations in principle can be solved, in practical calculations usually a 3-dimensional reduction is made through a quasi-potential approximation. We use the instantaneous/equal time framework [9], where it is assumed that the interaction depends only weakly on the relative-energy variable k_0. The t-matrix is generated using the Blankenbecler–Sugar reduction [10] in which the nucleons are treated in a symmetrical way. Using this assumption we may evaluate the interaction at the point $k_0 = 0$, thus effectively ignoring the retardation effects in the one-boson exchange.

The dynamics of the bremsstrahlung process is contained in the invariant matrix, $M_{fi} = \varepsilon_\mu \langle f|J^\mu|i\rangle$ with ε_μ the photon polarizations. The nuclear current is given by

$$\langle f|J_\mu|i\rangle = \langle f|\Gamma_\mu^{(1)}S^{(1)}T|i\rangle + \langle f|TS^{(1)}\Gamma_\mu^{(1)}|i\rangle$$

$$-i\langle f|TS^{(1)}\Gamma_{\mu}^{(1)}S_2T|i\rangle + (1 \leftrightarrow 2), \tag{4}$$

where $S^{(1)}, S_2$ are one- and two-nucleon propagators and T is the NN t-matrix. $|i\rangle$ and $\langle f|$ are the two-nucleon initial and final states, which are on the energy and mass shell. The NNγ vertex is given by

$$\Gamma_{\mu} = e\left(\gamma_{\mu} + \frac{i\kappa}{2m}\sigma_{\mu\nu}q^{\nu}\right), \tag{5}$$

where a possible dependence of form factors on the invariant mass of the off-shell nucleon has been ignored. The first two terms in Eq. (4) are the single-scattering contributions. The corresponding diagrams are shown in Figs. 1a and 1b, while in Fig. 1c is shown the rescattering diagram, corresponding to the third term in Eq. (4). The rescattering diagram is clearly a one-loop calculation and it involves an integration over the relative 4-momentum k. Since we have implicitly assumed in the equal-time approximation that the two-nucleon t-matrix does not depend on the relative-energy variable k_0, the k_0 integration contains only the singularities in the nucleon propagators and it can be done analytically by contour integration. There are three contributions, corresponding to each internal nucleon in the loop being on the mass shell (i.e. $(P_i+k)^2 = m^2$). One of these corresponds to the spectator contribution.

Due to the emission of the photon, we have in general to know the NN t-matrix in various different frames in order to calculate the bremsstrahlung amplitude. Usually the NN interaction is determined in the center-of-mass (c.m.) system of the nucleon pair. If we choose the c.m. system of the initial protons to calculate the amplitude, the t-matrix for diagrams involving the NN interaction after emission of the photon is obtained through the use of the Lorentz structure of the t-matrix

$$T(p',p;P) = \Lambda(\mathcal{L}) \times T^{\rm cm}(\mathcal{L}^{-1}p', \mathcal{L}^{-1}p; \mathcal{L}^{-1}P)\Lambda^{-1}(\mathcal{L}). \tag{6}$$

Here $\Lambda = \Lambda^{(1)}\Lambda^{(2)}$ is the spinor transformation for the boost $\mathcal{L}$ from the calculation frame to the c.m.-frame of the NN interaction. The boost is proportional to the photon momentum, and as a result can have substantial effects. In all calculations presented here these boost effects are taken into account.

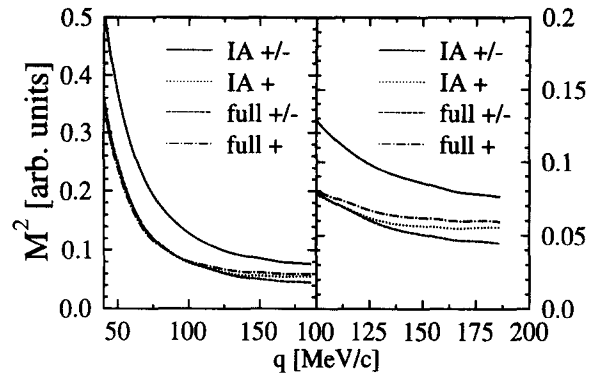

Fig. 2. Invariant-amplitude squared for fixed center-of-mass angles ($\theta_1 = 36°$, $\theta_2 = 69°$ and $\theta_\gamma = 124°$) and varying energy of the incoming proton, shown as a function of the photon momentum q. The right panel shows a magnification of the higher-momentum region. The lines labeled IA give calculation on the level of the Impulse Approximation with (full line, IA $+/-$) intermediate negative-energy states and without (dotted line, IA $+$). The lines labeled full give the calculations including the rescattering contribution with (dashed line, full $+/-$) and without (dash–dotted line, full $+$) intermediate negative-energy states.

In accordance with our qualitative discussion we indeed find that the contribution in the single-scattering diagrams from negative-energy states is significant for all photon momenta q. In Fig. 2 we compare a calculation of the invariant-matrix squared with intermediate negative-energy states included and without, for varying energy of the incoming proton in a kinematic situation with fixed angles $\theta_1 = 89°$, $\theta_2 = 69°$ and $\theta_\gamma = 124°$ in the c.m. system of the incoming protons. For all values of q we find that for the single-scattering diagrams contributions from negative-energy states lead to a 30% increase in the cross section as compared to only keeping the positive-energy states. Also shown in Fig. 2 are the results of a full calculation, that is, including the rescattering terms. The dashed line is a calculation including intermediate negative-energy states, and the dash–dotted line is the result if we only include positive-energy states. Adding the rescattering contributions clearly changes the results of the single-scattering amplitude with the complete nucleon propagator substantially. We find in particular, that the two full calculations coincide at low photon energies and are close to the single-scattering prediction, when only the contribution from positive-energy states is kept. This is in agreement with the findings of Ref. [4]. However, for photon momenta of the order of 100 MeV/c the calculations begin to show a clear difference, up to over 25% for 200 MeV/c.

We have been able to verify that the observed vanishing of the effects of negative-energy states for low photon momentum in the full calculation is a direct consequence of the low-energy theorem for NN bremsstrahlung [2], which states that for small photon momentum q the matrix element can be expanded as

$$M = \frac{A}{q} + B + Cq + \mathcal{O}(q^2). \tag{7}$$

The constants A and B can be shown to depend only on static properties of the proton, such as its charge and magnetic moment, and the on-shell t-matrix. To satisfy current conservation in calculating the invariant matrix one has to include both external contributions (where the photon couples to an external proton) and internal contributions. The effects of negative-energy states from the external diagrams in the constant B can be shown to cancel exactly with those from the internal diagrams. A similar result was recently obtained for the case of virtual bremsstrahlung [11]. In the present calculation, the external diagrams correspond to the single-scattering diagrams, and the internal diagrams correspond to the rescattering contributions. Therefore, only if we include the rescattering term the effects of negative-energy states vanish in the leading two orders of the low-energy limit. As shown at higher energies (beyond the validity of the low-energy limit) the effects of negative-energy states can be appreciable.

The cancellation at first seems counterintuitive, since in a perturbative treatment of the NN interaction the single scattering contribution appears to be of order V, whereas the rescattering contributions are of order V^2. To clarify the cancellation we have looked at the case of approximating the full t-matrix by the Born approximation. In lowest order of the potential V only the single-scattering diagrams contribute. In this case one can readily show analytically that the negative-energy state contributions cancel when the photon energy goes to zero. Aside from a partial cancellation between the diagrams with emission from particle 1 and those from particle 2 due to exchange symmetry, there is a cancellation of the negative-energy state contributions between diagrams with emission from the initial and final proton due to a difference in sign. Hence, from this we infer that the

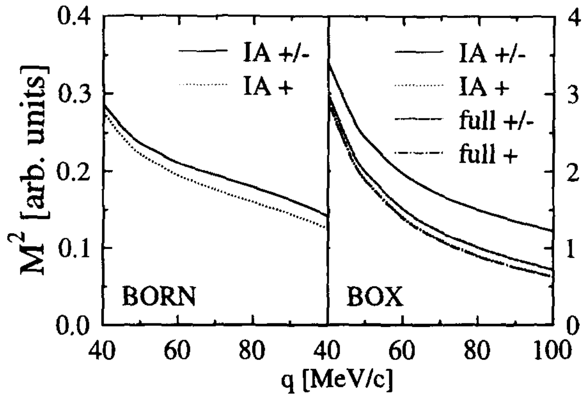

Fig. 3. Invariant-amplitude squared for fixed E_{lab} = 140 MeV and photon angle θ_γ = 170°. The left panel shows the Born result, the right panel shows a calculation with the box diagram. In the Born case there is no noticeable difference between a calculation including negative-energy state contributions (full line, IA +/−) and without (dotted line, IA +). In the case of the external box diagrams the difference between a calculation including negative-energy state contributions (full line, IA +/−) and without (dotted line, IA +) is large and in lowest order independent of q. Only if we include the internal diagrams (respectively dashed line, full +/− and dash-dotted line, full +) the negative-energy state contributions cancel.

large contribution from the negative-energy states to the single-scattering amplitude as found in the actual calculations is due to the higher-order contributions in V. This conclusion is in complete agreement with the actual numerical calculations with our relativistic OBE model using various approximations to the NN amplitude. In Fig. 3a we show the results when we take the Born approximation for the t-matrix in the kinematics with fixed proton energy E_{lab} = 140 MeV and photon angle θ_γ = 170° as a function of photon momentum q (through varying equal proton angles $\theta_1 = \theta_2$). To see the cancellation at a higher order in the interaction V we have calculated as an example the V^2 order contributions. The resulting NN t-matrix give rise to external box diagrams and an internal diagram where the emission of the photon is from within the box. In general, we do not expect that the contributions from negative-energy states in the diagrams with emission from the initial and final proton cancel. In view of the Ward identity, only after adding the diagram with emission from within the box the expected low-energy behavior should be recovered. The results for the box diagram calculations are plotted in Fig. 3b, where the kinematics is the same as for Fig. 3a. The full and dotted line are calculations without the internal diagram, respectively with and without negative-energy states between the NN and NNγ interaction. We clearly see that the deviation between the prediction with negative-energy states included and that without is roughly a constant, so that the negative-energy states clearly contribute to the constant B in Eq. (7). If we include the internal diagram we get the dashed line (full, $+/-$) for the calculation with negative-energy states and the dash-dotted line (full, $+$) for one with only positive-energy states. The negative-energy state contributions to the constant B cancel. This explicitly demonstrates that the cancellation is order-by-order in the strong coupling constant, as expected.

Using this relativistic NN force model, we may calculate the bremsstrahlung cross section and analyzing power for the case of the TRIUMF [12] kinematics. In Fig. 4 our predictions are shown together with the experimental data. The upper plots show the result at proton angles $\theta_1 = 12°$, $\theta_2 = 12.4°$ (left) and $\theta_1 = 28°$, $\theta_2 = 12.4°$ (right) for the cross section, while the lower plots are the results for the analyzing power at $\theta_1 = 14°$, $\theta_2 = 12.4°$ (left) and $\theta_1 = 28°$, $\theta_2 = 12.4°$ (right). The theoretical predictions for the complete calculation including both positive- and negative-energy states are given by the solid lines, while the dotted line is a calculation including only positive-energy states. The data do not include the normalization factor 2/3. The calculation with only positive-energy states included is in agreement with earlier calculations using realistic non-relativistic NN interactions such as the Bonn-PQ interaction used by Herrmann and Nakayama [13]. Our predictions at proton angles $\theta_1 = 12°$, $\theta_2 = 12.4°$ (left) of the differential cross section for example around the minimum are within a few percent the same as obtained in Ref. [13]. Comparing with the results including both rescattering contributions and relativistic spin corrections, which we automatically include since we use full 4-component spinors, we find some differences in the cross section results, at forward and backward photon angles. These differences are due to the details of the NN interaction. The inclusion of intermediate negative-energy states leads to a decrease in the cross section, particularly at forward and backward angles, where the photon momentum is highest and effects up to 20% are found. This is in agreement with the results

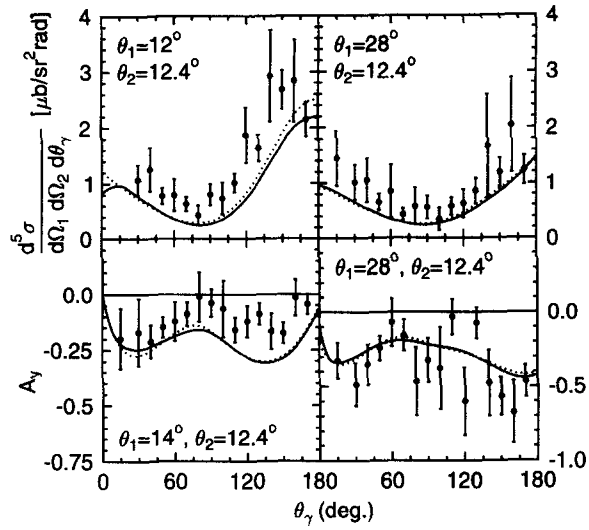

Fig. 4. Comparison of the calculated cross section (top) and analyzing power (bottom) to the experimental data from the TRIUMF experiment (with E_{lab} = 280 MeV). Two different kinematical situations are shown, left: $\theta_1 = 12°$ (14°), $\theta_2 = 12.4°$, right: $\theta_1 = 28°$, $\theta_2 = 12.4°$. The full line is a calculation including negative-energy states, the dotted line is with only intermediate positive-energy states.

reported by Eden et al. [14], who find a suppression of the cross section of the same order of magnitude, although the absolute values of their calculated cross section is considerably higher. This may be primarily due to the positive-energy state contribution, for which case their predictions deviate substantially from our and the non-relativistic model calculations.

The result for our present calculation is that the cross section is underpredicted, which is clearly not improved by including intermediate negative-energy states. At the same time the analyzing power is reasonably well reproduced. Some other authors [15,16] have given estimates for the contribution from meson-exchange currents and found them to be small in the kinematical regions under consideration. Including these contributions would not explain the difference, so that the discrepancy between the calculations and experiment remains.

In conclusion, we have shown that relativistic effects from contributions of negative-energy states of the proton are significant in proton–proton bremsstrahlung both on the level of single-scattering diagrams and in a full calculation. They are typically of the order of 10 to 20 percent in kinematical situations probed by the existing bremsstrahlung experiments and are found to become increasingly more important at higher photon energy. It should be noted that although the relativistic OBE interaction parameters used in our study have not been obtained through a χ^2 fit [7–9] to the NN data, the resulting phase shifts are in reasonable agreement with experimental ones, but not of the same quality as for the realistic non-relativistic potential models. In this connection we expect that a more refined relativistic NN interaction might change the absolute predictions somewhat, but that qualitatively the conclusions about the changes due to negative energy states as studied in this paper will remain the same. It is clearly of interest to study in more detail the model dependence of these relativistic effects.

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